

Economical evaluation of vehicles fleets' replacement considering potential environmental impacts resulting from carbon emissions

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Abstract

Nowadays the companies need to have efficient cost structures to be competitive, for that reason, the use of economic models to analyze the replacement process is very important and decisive. The acquisition of vehicles with lower CO₂ emissions is stimulated by governments that use legal frameworks to increase taxes related to the vehicles that have higher CO₂ emissions.

The contribution of this study is the definition of discounted cash-flows methodologies that use deterministic and stochastic environments to evaluate the replacement process considering the cash-flows related to vehicles and the legal framework related to taxes and rent-a-car companies. The most relevant variables to the replacement process are the ISV (vehicles tax), investment value, salvage value and the maintenance costs. The uncertainty related to the maintenance costs is considered for the stochastic environment. Other important input for the methodologies is the legal constraint of five years that says that the rent-a-car companies are obliged to have vehicles with not more than 5 years. This affects the maturity of the vehicles and is important to the replacement process. Considering the replacement of gasoline and diesel vehicles by hybrid vehicles there is the tendency to have fast vehicle's replacement and to choose replacing the vehicles in the first period. The CO₂ emissions related to the CO₂ legal framework and CO₂ taxes affect the replacement levels and are relevant to the replacement process.

Keywords: replacement process, vehicles, CO₂ emissions, CO₂ legal framework, deterministic, stochastic.

1 Introduction

In the today's world in which the markets show high levels of uncertainty and trough efficient cost structures it is essential that the companies create sustainable competitive advantages. Therefore, it is also fundamental that selling and buying decisions of vehicles should be based on economic analysis under uncertainty environments, similar to the ones that exist on the markets.

Nowadays, the search for sustainable economic solutions is stimulated by governmental organizations through a higher tax burden on products that use the environmental resources in a non-renewable way. Besides the financial aspect, initiatives such as, the Green Procurement Program in the European Union and the existence of Ecolabels that promote the products and equipment that operate in a sustainable way have been causing a change of paradigm in the

corporate procurement. In the same way, companies must foment economic appraisals for the asset replacement problem that integrate environmental variables to minimize costs and maximize the sustainability of those replacement operations (ECE, 2012).

The sector of Light Duty Vehicles represents 10% of CO_2 in the whole world (EESC, 2010). This evidence provides enough motivation for analyzing the environmental impacts of gas emissions (especially carbon emissions) on the asset replacement process. In 1997, an agreement was reached by 159 nations, where several goals were defined to promote the reduction of pollution gases emissions. This agreement was known as the Kyoto Protocol and it was created under the United Nations Convention on Climate Change. The objective of this Protocol is to establish compulsory goals for emissions reduction in 37 industrialized countries. They should, in average, reduce 5% below the 1990' levels, for a 5 years period, between 2008 and 2012. In order to fulfill the agreement, some countries defined additional measures where the goal was to reduce the carbon emissions on 25%, concerning 2006' values (EESC, 2010). This new goal also implied more severe penalties to old vehicles that showed high levels of CO_2 emissions.

2 Literature Review

2.1 Equivalent Annual Cost (EAC)

This method is used in deterministic environments. EAC methodology uses normally operating costs, maintenance costs, investment values and salvage values to calculate annual costs. To achieve Equivalent Annual Cost the method applies an interest rate to the Cash inflow/outflow in order to discount back to its present value (PV). Then the method transforms the Cash-Flows in an annuity value. The equipment that should be replaced is called the defender and the new equipment that should replace the old is called the challenger. The method compares EAC of the defender equipment with EAC of the challenger equipment for the n th periods (n). When the old equipment and the new equipment are the same, the optimal replacement timing is when EAC is minimum, but when the equipment are different (defender and challenger) the solution is more complex. To define a critical replacement timing the method needs to balance EAC of the challenger equipment and the EAC to maintain the defender equipment. The EAC tends to decrease with an higher economic lifetime of the equipment. (Riggs, Bedworth, & Randhawa, 1997)

2.2 Real Options Theory (ROT)

In deterministic environments, there is no uncertainty associated to the salvage value, to operating costs and maintenance costs. Economic and competitive interactions originate cash flows structures different from what was initially estimated. Therefore, discounted-cash-flow (DCF) approaches to vehicles' replacement have a gap because they cannot properly evaluate real economic conditions (Zambujal-Oliveira & Duque, 2010), (Amram & Kulatilaka, 1999). For that reason, Stochastic Dynamic Programming and Real Options Theory appeared. The objective of recent papers that studied these theories is to create practical and usable models in assets' replacement management (Adkins & Paxson, 2008), (Trigeorgis, 1996). The ROT's methods that are most used are the closed-form solutions, partial-differential equations, and the binomial lattice (Mun, 2002). The closed-form solutions are models of equations that can be solved given a set of input assumptions. They are precise, fast, and easy to implement with the knowledge of basic programming but can be difficult to explain because the stochastic calculus involved is complex. Closed-form solutions also tend to be specific so they have a low flexibility. Black-Scholes model - Black & Scholes (1973) - was one of the first closed-form solutions created to study the price of options. Partial-differential equations models are used to solve problems when the variables are associated to mathematical functions. ROT uses variables that are associated to probability distribution functions (uncertainty), because of that models such as partial-differential equations can be considered. Binomial lattices are based on binomial distribution and uses event trees to search for the best solution. They are easy to implement and to explain. Binomial lattices can be used in different problems because they are highly flexible. But two possible weaknesses are the significant computing power and time-steps

to obtain good approximations that are needed. There are two ways to apply Binomial lattices, risk-neutral approach and replicating portfolio approach. The results obtained by the use of binomial lattices tend to approach those derived from closed-form solutions (Mun, 2002), (Broyles, 2002). To use ROT an historical cost analysis must be performed in order to identify patterns and to assess input parameters. To generate paths for stochastic variables one of the most used methods is Monte Carlo - simulation technique based on repeated random sampling to compute their results. This outputs provided by MC simulations will permit to estimate the future cash-flows and will also be used to fill the Real Options Analyses (ROA) (Pridgen, 1968).

3 Replacement methodologies

3.1 Equivalent Annual Cost (EAC)

For this study, the propose objective is to determine the replacement of different vehicles There is the assumption that the replacement can occur after one year of utilization of the defender vehicle and that in the fifth year of utilization the company is obliged to replace the vehicle (legal constrain). The replacement procedure works in the following way (Brealey & Myers, 2006; Filho & Kopittke, 2007): the estimation of the challenger EAC considers a given maturity of five years. Thus, the vehicle will have an economic lifetime of five years, not considering a replacement before the fifth year (for the challenger option); primarily, the defender EAC is calculated considering the economic lifetime of the defender vehicle. In the first period, it considers one year of past operation and four more years of future operation. This means that in each period the defender EAC will be determined considering the free cash-flows to the firm of future operation (cash-flows to maintain the defender vehicle). For the defender vehicles the investment value considered in each period is the residual value of the vehicle. This happens because the residual value is the economic value that the rent-a-car company would receive if the vehicle was sold in that period. The replacement procedure compares the $EAC_{defender}$ with the $EAC_{challenger}$. It considers that the operational cash-flows of the defender increase along the time. The replacement procedure compares the EAC in each period (each period is related to the vehicle's age of the defender) - when the $EAC_{defender} > EAC_{challenger}$ the defender vehicle should be immediately replaced by the challenger vehicle and when $EAC_{defender} < EAC_{challenger}$ the defender vehicle shouldn't be replaced in that period. The first period when the $EAC_{defender}$ is higher than the $EAC_{challenger}$ is the critical timing of replacement. Equation 1 represents the $EAC_{defender}$ calculation:

$$EAC(xa) = ((SV(xa) \times (1 - t) + BV(xa) \times t))(A / P, i, n - xa) + \sum_{x=xa+1}^n \frac{(MC(x) + INS(x) + INSP(x) + IUC(x)) \times (1 - t) - t \times DC(x)}{(1 + i)^{x-xa}} (A / P, i, n - xa) - \frac{((SV(n) \times (1 - t) + BV(n) \times t))(A / F, i, n - xa)}{(1 + i)^{n-xa}} \quad (1)$$

A/P: Capital recovery factor; A/F: Sinking fund factor; i: Discount rate; n: Last period (n=5); xa: vehicle's age of the defender ($1 \leq xa \leq 4$); SV: Salvage value; BV: Book Value; MC: Maintenance cost; INS: Insurance cost; INSP: Inspection cost; DC: Depreciation costs; ISV, IUC: Taxes; t: Firm's average tax rate

Equation 2 represents the $EAC_{challenger}$ calculation:

$$EAC = ((I(0) + ISV(0))(A / P, i, n) + \sum_{x=1}^n \frac{(MC(x) + INS(x) + INSP(x) + IUC(x)) \times (1 - t) - t \times DC(x)}{(1 + i)^x} (A / P, i, n) - \frac{((SV(n) \times (1 - t) + BV(n) \times t)) \times (A / F, i, n)}{(1 + i)^n}) \quad (2)$$

A/P: Capital recovery factor; A/F: Sinking fund factor; i: Discount rate; n: Last period (n=5); I(0): Investment value (vehicles' price); SV: Salvage value; BV: Book Value; MC: Maintenance cost; INS: Insurance cost; INSP: Inspection cost; DC: Depreciation costs; ISV(0), IUC: Taxes; t: Firm's average tax rate; xa: vehicle's age of the defender ($1 \leq xa \leq 5$)

The $EAC_{challenger}$ included in equation 2 has always the same value because the maturity of the challenger vehicle is always equal to 5 years. The critical replacement level is the EAC related to the critical replacement timing. If the decision is to replace the defender vehicle the critical replacement level is the $EAC_{challenger}$ but if the decision is to don't replace the defender vehicle the critical replacement level is the $EAC_{defender}$ in the first period (EAC to maintain the defender vehicle until the last year).

3.2 Real Options Theory (ROT)

This topic explains the Real Options model used in this analysis to evaluate vehicles' replacement. The real options model was created using as reference (Copeland & Antikarov, 2001). The goal of this model is to find a feasible approach to evaluate uncertainty considering historical data related to the utilization of a vehicle (defender vehicle). The data related to the challenger vehicle is considered deterministic because it is considered that the challenger vehicle has never been used before. This study will consider uncertainty related to the variable maintenance costs. This means that the variable maintenance costs will have a probability distribution in each period that will be used as input of the Monte Carlo simulation. The FCFF related to the defender vehicle will be used to define a binomial tree. The Risk neutral approach will be used in this analysis. To find the expected volatility of the project this study will use the standard deviation of the variable z (equation 3) considering uncertainty related to the maintenance costs and the outputs of the Monte Carlo simulation.

$$z = \ln \left(\frac{PV_1 + FCFF_1}{PV_0} \right) \quad (3)$$

z : capital cost with uncertainty; PV_0 : Present value in period zero (considering five more years of future operation); PV_1 : Present value in the first period (considering one year of past operation and four more years of future operation); $FCFF_1$: Free Cash Flows in period one

The following procedure will be used to evaluate the replacement process considering Real Options:

First step - to build a binomial tree related to the values of the uncertainty variable (maintenance costs). To do that we will use the volatility related to the uncertainty variable to produce the "up movement" and "down movement". Using the uncertainty variable value in the first period and multiplying this value by the "up movement" and "down movement" we will determine in each period the "up state" and "down state" of the tree – see equation 4 and 5.

$$u = e^{\sigma\sqrt{T}} \quad (4)$$

$$d = 1/u \quad (5)$$

u : Up movement; d : Down movement; σ : Volatility; T : length of time between nodes

Second step - Using the deterministic values (taxes, insurance costs, inspection costs, depreciation costs and salvage value) and the values defined before (uncertainty variable values for the "up state" and "down state" of the maintenance costs we can fill each node with the FCFF related to the defender vehicle. It's important to understand that the investment value is not yet considered and that the salvage value is only considered in the last period of the tree.

Third step - Using the FCFF tree (see topic 2) we will start from the last node to recombine the tree using the "up probabilities - p " and "down probabilities - $(1-p)$ ". These probabilities will be determined using the project volatility (cost of capital volatility – standard deviation of z) and the RNA approach. Equation 6 shows the calculation for each node of the tree. The last nodes maintain the same values – FCFF in the last period.

$$V_{(s,t)} = \frac{(V_{(s,t+1)} + FCFF_{(s,t+1)}) \times p + V_{(s+1,t+1)} + FCFF_{(s+1,t+1)} \times (1-p)}{(1+r_f)} \quad (6)$$

$$\text{Where: } p = \left(\frac{(1+r_f) \cdot d}{u-d} \right)$$

$$(1-p) = \left(\frac{u - (1+r_f)}{u-d} \right)$$

s : Node – $s \geq 0$; t : Period – $t \geq 1$; r_f : Risk-free rate; $FCFF$: Free cash-flow to the firm; p : RNA probability; V : Value of the future cash-flows

Fourth step - The values of the future free cash flows to the firm that were defined above (see third topic) will be converted into an EAC. This is necessary because we want to compare vehicles with different maturities. The investment value related to defender vehicle in each period (residual value) will also be considered before the value of the future cash-flows is converted into an EAC..

Fifth step - The procedure that will be used to evaluate the decision of replacement is to compare the $EAC_{\text{challenger}}$ considering the deterministic analysis and the EAC_{defender} considering

uncertainty (EAC of the fourth step) in each node. The replacement procedure defines that when the $EAC_{\text{defender}} > EAC_{\text{challenger}}$ the defender vehicle should be immediately replaced by the challenger vehicle and when $EAC_{\text{defender}} < EAC_{\text{challenger}}$ the defender vehicle shouldn't be replaced.

4 Case study analysis

4.1 Definition

To study the replacement process it was created a case study that is an academic example about vehicles' replacement. There were defined three vehicles: Toyota Auris 1.6 (gasoline vehicle), Toyota Auris 1.8 (hybrid vehicle - gasoline + battery energy) and Toyota Auris 2.0 (diesel vehicle). There are two possible options for the replacement: replace the vehicle Toyota Auris 1.6 for Toyota Auris 1.8 and to replace the vehicle Toyota Auris 2.0 for Toyota Auris 1.8. The vehicles Toyota Auris 1.6 and Toyota Auris 2.0 are considered as the defenders vehicles and the vehicle Toyota Auris 1.8 is considered as the challenger vehicle. In order to maintain similar vehicles performance and to create feasible/realistic replacements there were chosen three vehicles with the same brand (Toyota) and model (Auris).

For their activity, the rent-a-car companies need to know the replacement timing and level for each replacement option and to understand the effect of environmental legal framework in the replacement process. Toyota Auris 1.6 has 153g/km of CO₂ emissions and 1598 cm³ (gasoline) of engine size, Toyota Auris 1.8 has 89g/km of CO₂ emissions and 1798 cm³ (gasoline + hybrid) of engine size and Toyota Auris 2.0 has 138g/km of CO₂ emissions and 1998 cm³ (diesel) of engine size.

4.2 Maintenance costs

The estimated maintenance costs of the vehicles (defined in the case study) correspond to a 1 year of operation or 15,000 km traveled (Toyota, 2012). To define increasing maintenance costs this study will use aggregate maintenance costs. There is also the assumption that each vehicle of the case study travels 15,000 km per year. To predict future values on a deterministic environment this study used a linear regression. Equation 7, 8 and 9

$$MC_{\text{Auris 1.6}} = 163.46x \text{ with } x \geq 0 \quad (7)$$

$$MC_{\text{Auris 1.8}} = 167.05x \text{ with } x \geq 0 \quad (8)$$

$$MC_{\text{Auris 2.0}} = 215.95x \text{ with } x \geq 0 \quad (9)$$

MC: Maintenance cost; x: Period (year)

formalizes the maintenance costs for the vehicles.

4.3 Inspection and Insurance costs

The first scheduled inspection is 4 years after the first vehicle registration. After the first inspection the vehicle must be inspected once every 2 years. After the 8th year of registration the inspection must be annually performed. The inspection cost is 30.54€ according the Portuguese law (Decreto-Lei nº 144/2012, 2015).

For estimating insurance costs, this analysis considered a simulation with eleven types of scenarios: date of birth, date of driving license, number of years without accidents, number of accidents in the last 5 years (Império Bonança, 2012). The values per year that were computed are: 234.05€ for Auris 1.6; 273.14€ for Auris 1.8 and 273.14€ for Auris 2.0.

4.4 Salvage value

The investment values (vehicle's price) used in the analysis comes from a list of acquisition prices provided by Toyota (Toyota, 2012). To estimate the salvage value, the analysis used estimated market values that were established by vehicles' experts which constituted references to the market buyers and sellers. These values are determined using vehicle's dealerships information about the vehicles that were sold. They consist on an average price considering that information (Guia do Automóvel, 2012; Toyota, 2012). This study considers an exponential regression in order to predict a salvage value equation. Equation 10, 11 and 12 formalizes the salvage for the vehicles. These values include ISV and VAT.

$$SV_{Auris\ 1.6} = 25030 e^{-0.165x} \text{ with } x \geq 0 \quad (10)$$

$$SV_{Auris\ 1.8} = 25242 e^{-0.169x} \text{ with } x \geq 0 \quad (11)$$

$$SV_{Auris\ 2.0} = 31951 e^{-0.208x} \text{ with } x \geq 0 \quad (12)$$

SV: Salvage value; x: Period (year)

4.5 Vehicles tax (ISV)

Vehicles tax (ISV) is a tax paid only one time and after the vehicle registration. It evaluates two elements: CO₂ emissions (environmental impact) and engine size (CISV - Decreto de Lei nº 82-D/2014, 2015).

The ISV (Engine Size) is calculated using equation 13:

$$ISV(Es) = Es \times RtEs - CfEs \quad (13)$$

Es: Engine size; RtEs: Rate per engine size; CfEs: Correction factor of Engine size

The ISV (CO₂ emissions) is calculated using equation 14:

$$ISV(CO_2e) = CO_2e \times RtCO_2e - CfCO_2e \quad (14)$$

CO₂e: CO₂emissions; RtCO₂e: Rate per CO₂ emissions; CfCO₂: Correction factor of CO₂emissions

For diesel vehicles with an emission of particles higher than 0.002 g/km the ISV has an increase of 500 €. The total ISV is presented by equation 15 (CISV - Decreto de Lei nº 82-D/2014, 2015):

$$TISV = ISV(CO_2e) + ISV(Es) \quad (15)$$

TISV: Total ISV

The ISV is 3 718.73€ for Auris 1.6; 1 852.76€ for Auris 1.8 and 7 876.73€ for Auris 2.0.

4.6 Circulation Tax (IUC) and Value added tax (VAT)

IUC is an annual tax. For new vehicles the tax can be paid 90 days after the vehicle registration. It evaluates the same two elements: CO₂ emissions (environmental impact) and engine size (CIUC, Tax code, 2015). IUC has also a coefficient that is related to vehicle's age (CIUC - Decreto de Lei nº 82-B/2014, 2015) – 1 for vehicles acquired in 2007, 1.05 for vehicles acquired in 2008, 1.10 for vehicles acquired in 2009 and 1.15 for vehicles acquired after 2010. The total IUC is represented by equation 16:

$$TIUC = Coef \times (IUC(Es) + IUC(CO_2e)) \quad (16)$$

TIUC: Total IUC; Coef: Coefficient

The Value added tax falls upon the ISV plus the vehicle price. The tax rate is 23% (Government Budget - Lei nº 82-B/2014, 2015). The IUC is 164.51€ for Auris 1.6; 196.25€ for Auris 1.8 and 249.48€ for Auris 2.0.

4.7 Taxes` reduction for rent-a-car companies

For rent-a-car companies the ISV has a discount of 40% if the vehicle has an hybrid engine or if it has a level of CO₂ emissions lower than 120g/km (CISV - Decreto de Lei nº 82-D/2014, 2015).

4.8 Replacement constraint for rent-a-car companies

In legal terms, Portuguese rent-a-car companies are obliged to have vehicles with no more than 5 years after the first vehicle's registration (Decreto-Lei n.º 207/2015, 2015).

4.9 Depreciation costs

For estimating the depreciation costs generated by the vehicle, this study considered "Straight-line method" with a depreciation rate of 25% (RAD, 2009), meaning that after 4 years of operation, the vehicle will be fully depreciated (booking value equal to 0). The depreciation costs do not consider VAT because this tax is deductible for income tax purpose.

4.10 Weighted average capital cost

The real WACC considered in this study was 2.9%. This value was computed considering a risk-free rate of 1.6% (Investing.com), a market return rate of 5.0% (DBK, 2014), a beta of 0.58 (Investing.com), a cost of debt of 4.6% 2016 (Portuguese National Bank, 2015), an average debt value for the rent-a-car market of 4 346 267.72€, an average equity value for the rent-a-car market of 900 808.78€ (INE, 2015), a firm`s average tax rate of 21% (Government Budget - Lei nº 82-B/2014, 2015) and an inflation rate (2015) of 0.7% (Government Budget - Lei nº 82-B/2014, 2015)

5 Replacement process analysis

5.1 EAC considering CO₂ emissions legal framework

The graphical analysis of EAC (for each defender vehicle and for the challenger vehicle) can be seen on the following graphic (see figure 1).

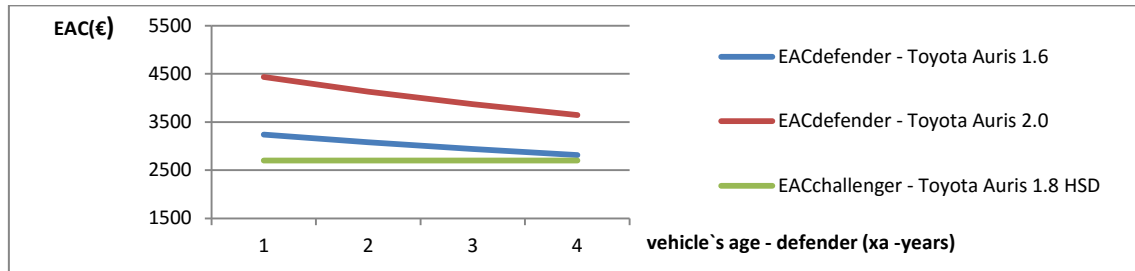


Figure 1 - EAC for defender and challenger vehicles

The $EAC_{challenger}$ is equal in each period because its maturity is always the same (5 years). In the first period the $EAC_{defender} (xa=1)$ (for both defender vehicles) $> EAC_{challenger} (xa=1)$, meaning that in the first period (vehicle's age of the defender equal to one), the best decision is to replace the defender vehicle for the challenger vehicle. For that reason the critical replacement timing is one year and the critical replacement level is 2702.41€. Comparing Toyota Auris 1.8 (challenger) with Toyota Auris 2.0 (defender) the $EAC_{challenger}$ in each period is always lower than $EAC_{defender}$. This means that, even if the decision about replacement is delay, the replacement decision in each period is to replace the defender vehicle. Comparing Toyota Auris 1.8 (challenger) with Toyota Auris 1.6 (defender) in each period, the $EAC_{challenger}$ is also always lower than $EAC_{defender}$. Figure 1 supports the $EAC_{defender}$ tendency to decrease along the time, with this tendency there are only two possible outcomes for the critical replacement timing: replace the defender vehicle in the first period or don't replace it.

5.2 EAC without considering CO₂ emissions legal framework

Figure 2 shows the EAC without CO₂ legal framework (for each defender vehicle and for the challenger vehicle). It excludes taxes related to CO₂ emissions and the CO₂ emissions legal framework - ISV (CO₂ emissions) including the 40% discount of ISV (related to CO₂ emissions) and IUC (CO₂ emissions).

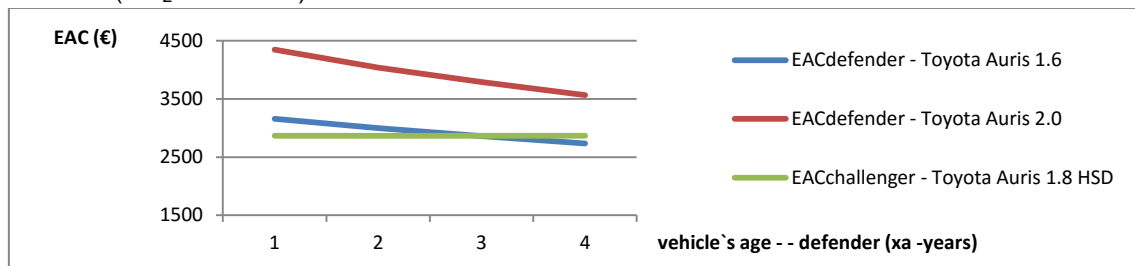


Figure 2 - EAC for defender and challenger vehicles without considering CO₂ legal framework

Comparing the contents of Figure 2 and Figure 1 (EAC table considering legal framework related to CO₂ emissions) the critical timing of replacement remains the same, the first period ($xa=1$ year). However, there is evidence of an increase of 6.0% of the critical replacement level. The difference between $EAC_{challenger}$ and the $EAC_{defender}$ is lower. In the third period and last period the EAC of Toyota Auris 1.6 (defender vehicle) is lower than the EAC of Toyota Auris 1.8 (challenger vehicle). This means that considering Toyota Auris 1.6 as the defender vehicle the critical timing of replacement is the first period but if the decision about replacement is delay until the third period the replacement decision is to don't replace the defender vehicle. Considering Toyota Auris 2.0 as the defender vehicle the conclusion is the same of topic 5.1 (EAC considering CO₂ emissions legal framework).

5.3 Real Options considering CO₂ emissions legal framework

To analyze the process of replacement considering a stochastic environment, this study computed a Monte Carlo simulation to predict the values of the variables with uncertainty for both defender vehicles. The chosen uncertainty variable is the maintenance costs and the cost

of capital rate is the risk-free (risk neutral approach). The first goal of the Monte Carlo simulation is to estimate the expected volatility of the project. Initially for the maintenance costs it was considered a lognormal distribution, with a standard deviation of 30% (30% of the average value) in each period and an average equal to the deterministic value in each period (assumption used to create inputs to Monte Carlo simulation). The lognormal distribution was chosen because it is the distribution probability most used to evaluate costs. This happens because combinations of lognormal distribution are themselves lognormal and the values are always positive (Copeland & Antikarov, 2001). There is also the assumption that the distribution probabilities between the periods are not correlated. There were generated 10,000 iterations for the maintenance costs in each period that originated 10,000 results for the variable z - z is considered as the cost of capital considering uncertainty. The maintenance costs volatility that is going to be considered is the standard deviation of the maintenance costs along the time period (5 years). One of the outputs of the Monte Carlo simulation were 10,000 sets of values for the maintenance costs along the time period, this means that we will have 10,000 standard deviations (volatilities) for each set of trials. The frequency of z per trial is shown in Figures 3 and 4.

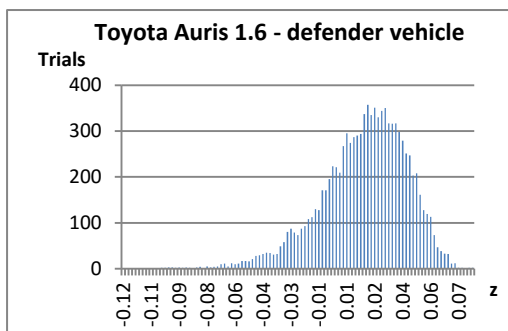


Figure 3 - Frequency of z for Toyota Auris 1.6

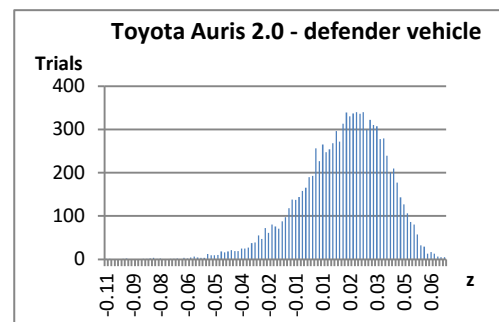


Figure 4 - Frequency of z for Toyota Auris 2.0

The standard deviation of z (project volatility) that was computed (see Figures 3 and 4) was 3.1% for Auris 1.6 and 4.5% for Auris 2.0 with a mean of 1.6% (risk-free-rate). For each defender vehicles (Toyota Auris 1.6 and Auris 2.0) the volatility computed was 0.48 with a standard deviation of 0.14. With these values it's possible to determine the values of EAC_{defender} with uncertainty minus the $EAC_{\text{challenger}}$ (deterministic value) in each node. The yellow cells are the positive values (nodes where the defender vehicle should be replaced). In the negative values ($EAC_{\text{defender}} < EAC_{\text{challenger}}$) the defender vehicle shouldn't be replaced – see Figures 5 and 6.

Toyota Auris 1.6: $EAC_{\text{defender}} - EAC_{\text{challenger}}$ (€)			
Vehicle's age (years)			
1	2	3	4
			274
		250	
	286		-186
386		-86	
	38		-362
		-215	
			-429

Figure 5 - EAC defender minus EAC challenger for Toyota Auris 1.6

Toyota Auris 2.0: $EAC_{\text{defender}} - EAC_{\text{challenger}}$ (€)			
Vehicle's age (years)			
1	2	3	4
			1072
		1084	
	1204		501
1453		681	
	918		283
		527	
			199

Figure 6 - EAC defender minus EAC challenger for Toyota Auris 2.0

The binomial trees of Figures 5 and 6 show that the critical replacement timing occurs in the first period because in the indicated period, the EAC_{defender} is always higher than the $EAC_{\text{challenger}}$. For Toyota Auris 1.6, in the third and fourth periods (last one) there are nodes with negative values (white cells) and positive values (yellow cells). This means that the uncertainty related to the maintenance costs produces replacement outputs that are not consistent and don't produce unique solutions about the replacement problem in those periods.

The values between $EAC_{defender}$ and $EAC_{challenger}$ became closer along the time of replacement, meaning that there are serious doubts about the replacement decision. Thus, a light variation on the data can change the outcome of the model. Supporting this evidence, we can verify that in the first period, the $EAC_{defender} - EAC_{challenger}$ has the higher value of all the periods, signifying that an early decision will maximize the difference between the $EAC_{defender}$ and the $EAC_{challenger}$.

5.4 Real Options without considering CO₂ emissions legal framework

To analyze the replacement of the vehicles considering uncertainty and excluding taxes related to CO₂ emissions and the CO₂ emissions legal framework. The project volatility excluding the effect of CO₂ emissions is 3.0% for Toyota Auris 1.6 and 4.3% for Toyota Auris 2.0. Figures 7 and 8 show $EAC_{defender}$ with uncertainty minus the $EAC_{challenger}$ (deterministic value) excluding the effect of CO₂ emissions. The yellow cells are the negative values (nodes where the defender vehicle should be replaced).

Toyota Auris 1.6: $EAC_{defender} - EAC_{challenger}$ (€)				
Vehicle's age (years)				
1	2	3	4	
			50	
		26		
	61		-413	
160		-314		
	-190		-590	
		-444		
			-658	

Figure 7 - EAC defender minus EAC challenger for Toyota Auris 1.6

Toyota Auris 2.0: $EAC_{defender} - EAC_{challenger}$ (€)				
Vehicle's age (years)				
1	2	3	4	
			927	
		938		
	1057		353	
1304		532		
	767		133	
		376		
			49	

Figure 8 - EAC defender minus EAC challenger for Toyota Auris 2.0

Considering Figures 7 and 8 the replacement of the defender vehicle for the challenger vehicle should occur in the first period (critical replacement timing). Comparing Figures 6 and 7 (values considering CO₂ emissions) and Figures 7 and 8 (values without considering CO₂ emissions), we can verify that the difference between $EAC_{defender}$ and $EAC_{challenger}$ is lower. The ISV reduction of 40% is the most relevant parameter for this difference. In what respect to Figure 7, we can find nodes with negative values (white cells) and positive values (yellow cells) in the second, third and fourth periods, meaning that the binomial approach related to the maintenance costs with uncertainty doesn't produce unique solutions about the replacement problem in those periods.

6 Sensitivity analysis

Table 1 summarizes the sensitivity analysis considering different variables, the EAC sensitivity and changes in the critical replacement timing.

Table 1 - Sensitivity analysis - *considering a time period of 4 years; *considering a time period of 6 years; *considering the difference of EAC with and without the 40% discount (hybrid vehicles)**

Variables	EAC sensitivity			Critical replacement timing
	Defenders		Challenger	
	Auris 1.6	Auris 2.0	Auris 1.8	
WACC	high	High	very high	doesn't change
Higher time period*	high (EAC increase)	high (EAC increase)	Low (EAC decrease)	doesn't change
Lower time period **	high (EAC decrease)	high (EAC decrease)	very low (EAC increase)	doesn't change
ISV - 40% discount***	equal	Equal	High	doesn't change
Investment (challenger)	equal	Equal	very high	change with variation higher than 10%
Salvage value (last year - challenger)	equal	Equal	High	change with variation higher than 20%
Maintenance costs (challenger)	equal	Equal	Low	doesn't change
Salvage value (last year - defender)	high	High	Equal	change with variation higher than 20%
Maintenance costs (defender)	Low	Low	Equal	doesn't change

With table 1 it's possible to verify that the investment value/ISV and salvage value are relevant to the critical replacement level. The maintenance costs are not relevant to the critical replacement level but are important to the replacement process (are relevant if the decision about replacement is delay after the first period).

Considering Real Options a lower volatility for the maintenance costs produces a lower EAC_{defender} minus $EAC_{\text{challenger}}$ value and an higher volatility for the maintenance costs produces an higher EAC_{defender} minus $EAC_{\text{challenger}}$ value but the critical replacement timing remains the same (first period).

7 Conclusion

This document shows the possibility to define suitable models to evaluate replacement timing and level of vehicles from the firm's point of view (rent-a-car). Considering the existing legal framework, the models consider the environmental impacts in the replacement process. In that way, the document defined two models of analysis (Equivalent Annual Cost and Real Options) in order to study the replacement problem considering two different environments: a deterministic environment and a stochastic (uncertain) environment. The data used to fill the models of this study came from different sources (Toyota, Guia do Automóvel, Império Bonança, legal framework – CISV and CIUC) The case study was composed using market data with the aim to perform a formal evaluation in order to create a set of outputs from different methodologies. The main goal of this study was to define an economic model to analyze non-identical replacements of vehicles belonging to a rent-a-car company. Another relevant goal was to understand the impact of CO₂ emissions on the replacement timing and replacement level. This study concluded that taxes and legal framework related to CO₂ emissions benefit the hybrid vehicle (challenger vehicle). According this study, the ISV tax reduction produces a motivation for replacement and the ISV tax reduction of 40% analyzed in this study supports that conclusion. According Portuguese law the ISV reduction happens for electric, hybrid vehicles and vehicles with CO₂ emissions lower than 120g/km. This study also shows that the taxes related to CO₂ emissions and the legal framework related to CO₂ emissions are more relevant to the replacement timing if the decision about replacement is delay after the first year. The output of the models that were created shows that there is the tendency to have a fast replacement. The study shows that the tendency is to replace the vehicle in the first period (first year – vehicles' age equal to one year). Rent-a-car companies also perform fast replacements for their fleets. Two of the biggest rent-a-car companies operating in Portugal - Avis e Europcar - perform average replacement timings of 6 months to 1 year (Europcar Portugal, 2015; Avis Portugal, 2015). Another conclusion that it's possible to extract from the models that were used is that if we delay the decision about replacement the values between EAC_{defender} and $EAC_{\text{challenger}}$ became closer in some cases and for a specific data. This means that if we decide to wait the tendency is to have more doubt about the decision and a lightly variation of the data can change the outcome of the model. The Real Options model produced similar results comparing with the EAC model. The Real Options model was important to analyze the case study considering uncertainty related to the maintenance costs and to verify the consistence of the results produced by the deterministic model (EAC). The MC is a variable that is not relevant to the decision of critical timing but it is relevant if the decision about replacement is delay after the first period, because of that the MC is considered as an important variable in the replacement process. The depreciation cost is also very important. A cost structure with higher depreciation costs tends to have lower EACs. A faster depreciation produces faster replacements and this information can be used to benefit an option that has lower CO₂ emissions. Faster depreciations for vehicles with lower CO₂ emissions and slower depreciations for vehicles with higher CO₂ emissions can produce a positive impact in the replacement decisions.

Bibliography

Adkins, R., & Paxson, D. (2008). *An Analytical Real Option Replacement Model with Depreciation*. Manchester: University of Manchester.

- Amram, M., & Kulatilaka, N. (1999). *Real Options. Managing Strategic Investments in a Uncertain World* (1st ed.). Boston: Harvard Business School Press.
- Avis Portugal. (2015). Retrieved Dezembro 15, 2015, from <http://www.avis.com.pt/faqs>
- Bank of Portugal. (2015). *Mensal report - September 2015*.
- Barbir, F. (2007). *Sustainable energy production and consumption: benefits, strategies and Environmental Costing*. Naples: Springer.
- Bellman, R. E. (1955, Setembro). Equipment Replacement Policy. *Journal of the Society for Industrial and Applied Mathematics*, vol. 3, n.3, pp.133-136.
- Beveridge, G. S., & Schecht, R. (1970). *Optimization theory and practice*. Tokio: McGraw-Hill Kogakusha.
- Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy*, vol. 81, n.3, pp. 637-654.
- Blank, L. T. (2014). *Basics of engineering economy*. McGraw-Hill.
- Borgert, A., Hunttemann, E. S., & Schultz, C. A. (2006). *Custo anual uniforme equivalente (CAUE) aplicado à avaliação de veículos populares*. Fortaleza: XXVI ENEGEP.
- Brealey, R. A., & Myers, S. C. (2006). *Principles of Corporate Finance* (6th ed.). New York: McGraw-Hill.
- Broyles, J. E. (2002). *Financial management and real options* (1st ed.). United Kingdom: Wiley.
- Copeland, T., & Antikarov, V. (2001). *Real Options: A practitioner's Guide* (1st ed.). New York: Texere.
- (2015). *Corporate Tax rates*. Deloitte.
- Damodaran, A. (2005). *Estimating Beta*. New York: NYU Stern School of Business.
- Damodaran, A. (2015). *Damodaran online*. Retrieved from <http://pages.stern.nyu.edu/~adamodar/>
- DBK. (2014). *Renting - Portugal*.
- Decreto de Lei nº 82-D/2014 de 31 de Dezembro, CISV 2015. Diário da República nº 252 (2014), 2.º Suplemento, Série I. Acedido a 1 Setembro de 2015. Disponível em www.portaldasfinancas.gov.pt
- Decreto de Lei nº 82-B/2014 de 31 de Dezembro, CIUC 2015. Diário da República nº 252 (2014), 1º Suplemento, Série I. Acedido a 1 Setembro de 2015. Disponível em www.portaldasfinancas.gov.pt
- Decreto de Lei nº 4/2015 de 22 de Abril, Regime das depreciações e amortizações. Diário da República nº 4 (2015), Série I. Acedido a 1 Setembro de 2015. Disponível em www.portaldasfinancas.gov.pt
- Decreto de Lei nº 207/2015 de 24 de Setembro de 2015, Diário da República nº 187 (2015), Série I. Acedido a 1 Outubro de 2015. Disponível em www.dre.pt
- Decreto de Lei nº 144/2012 de 6 Julho, Diário da República nº 140 (2012), Série I. Acedido a 1 Setembro de 2015. Disponível em www.dre.pt
- Decreto de Lei nº 82-B/2014 de 31 de Dezembro, Orçamento de Estado 2015. Diário da República nº 252 (2014), 1º Suplemento, Série I. Acedido a 1 de Agosto. Disponível em www.portaldasfinancas.gov.pt
- Europcar Portugal. (2015). Retrieved Dezembro 15, 2015, from <https://www.europcar.pt/b2b/frota>
- Feldens, A. G., Muller, C. J., Filomena, T. P., Neto, F. J., Castro, A. d., & Anzanello, M. J. (2010, Janeiro/Abril). Política para Avaliação e Substituição de Frota por Meio da Adoção de ABCustos Associação Brasileira de Custos, vol. 1, pp. 1-27.
- Filho, C., & Kopittke, B. (2007). *Análise de investimentos – matemática financeira, engenharia econômica, tomada de decisão e estratégia empresarial* (10th ed.). S. Paulo: Atlas.
- Guia do Automóvel. (2012, Março). Guia do Automóvel. *Guia do Automóvel*, 63-258.
- Guthrie, G. (2009). *Real Options in Theory and Practice* (1st ed.). New York: Oxford University Press.
- Hastings, N. A. (1968). *Some Notes on Dynamic Programming and Replacement*. Birmingham: Royal Military College of Science.
- Hynes, C. D. (2005). *Cost-Benefit Analysis of Aircraft Design for Environment using a Fleet Perspective and Real Options*. Massachusetts: Massachusetts Institute of Technology.
- Império Bonança. (2012). *Império Bonança*. Retrieved 2012, from Imperio Bonança - simulador de seguros: <http://www.meuportalfinanceiro.pt/simulador-de-seguro-automovel-imperio-bonanca/>
- INE. (2015). *Instituto Nacional de Estatística*. Retrieved from www.ine.pt
- Investing.com*. (n.d.). Retrieved 2015, from Investing: <http://www.investing.com/>

- Ismail Erol, W. G. (2003). A methodology for selection problems with multiple conflicting objectives and both qualitative and quantitative criteria. *International Journal of Production Economics*, vol. 86, pp. 187-199.
- Ji, D., & Kite-Powel, H. L. (1999). Optimal fleet utilization and replacement. *Transportation Research Part E: Logistics and Transportation Review*, vol. 36, n.1, pp. 3-20.
- Keles, P., & Hartman, J. C. (2004). Case study: bus fleet replacement. *Engineering Economist*, vol. 49, pp. 253.
- Marques, G. M., Silva, M. L., Leite, H. G., & Fontes, A. A. (2005). Aplicação da programação dinâmica na substituição de equipamentos. *Revista Árvore*, vol. 29, n.5, pp. 749-756.
- McArthur, A. T. (1975). *Dynamic programming applied to animal replacement decisions*. Canterbury: University of Canterbury.
- Minardi, A. (2004). *Real options theory applied to investment projects* (1st ed.). São Paulo: Atlas.
- Mun, J. (2002). *Real Options Analysis: Tools and Techniques for valuing strategic investments and decisions* (2nd ed.). USA: Wiley Finance.
- Portuguese National Bank. (2015, Março). *Boletim Estatístico Março 2015*.
- Pridgen, E. V. (1968). *Risk analysis of equipment replacement policies using Monte Carlo techniques* (Vol. 1st). Pennsylvania: University of Pennsylvania.
- RAD. (2009). *Decreto Regulamentar nº25/2009 – Regime das Amortizações e Depreciações* – Portuguese depreciation and amortization procedure.
- Riggs, J., Bedworth, D., & Randhawa, S. (1997). *Engineering Economics*. New York: McGraw-Hill.
- Roy, B., & Vincke, P. (1983). Multicriteria analysis: survey and new directions. *European Journal of Operational Research*, vol.8, pp. 207-218.
- Sepulveda, J. A., Souder, W. E., & Gottfried, B. S. (1984). *Schaum's outline of Theory and problems of Engineering Economics* (1st ed.). New York: McGRAW- HILL.
- Smokers, R., Vermeulen, R., Mieghem, R. v., Gense, R., Skinner, I., Fergusson, M., et al. (2006). *Review and analysis of the reduction potential and costs of technological and other measures to reduce CO2-emissions from passenger cars*. Netherlands: ECE.
- Sykes, A. O. (1992). *An Introduction to Regression Analysis*.
- Toyota. (2012). *Toyota - Catálogos online*. Retrieved from Toyota online: www.toyota.pt
- Trigeorgis, L. (1996). *Real options: managerial flexibility and strategy in resource allocation* (1st ed.). Massachusetts: Library of Congress Cataloging-in-publication Data.
- Valverde, S. R., & Resende, J. L. (1997). Substituição de Máquinas e Equipamentos: Métodos e Aplicações. *Revista Árvore*, vol. 21, n. 3, pp. 353-364.
- Vey, I. H., & Rosa, R. M. (2003). *Substituição de Frota em Empresa de Transporte Municipal de Passageiros: Um Estudo de Caso*. São Paulo.
- Waddell, R. (1983). A Model for Equipment Replacement Decisions and Policies. vol. 13, n. 4, pp. 1-7.
- Wade, M., & Boman, B. (2003). *Economic Considerations for Florida Citrus Irrigation Systems*. Florida: University of Florida.
- Zambujal-Oliveira, J., & Duque, J. (2010). Operational asset replacement strategy: A real options approach. *European Journal of Operational Research*, n. 210, pp. 318-325.